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MODELS OF DYNAMIC OBJECTS WITH SIGNIFICANT NONLINEARITY BASED ON TIME-DELAY NEURAL NETWORKS

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Abstract. The paper is devoted to the problem of nonlinear modeling of objects based on dynamic neural networks. The aim of the work is to improve the accuracy of modeling dynamic objects with significant nonlinearities using neural network models and to determine the scope of effective application of these models. This aim is achieved using time-delay neural networks. To assess the applicability of the proposed neural network models, the study considers simulation objects with two types of nonlinearities: smooth and piecewise linear (saturation). The investigation of suggested models accuracy in nonlinear dynamic object modeling involves two experiments: the study of the models' scalability with different input signals; the study of their extrapolation capabilities. The results of both experiments are compared with the modeling results using the compensatory method of deterministic identification based on functional series. The results of the experiments reveal that the suggested neural network models are not invariant concerning the input signal. However, when trained on a sufficient amount of data generated from input signals of the same type as in the test data set, these models can effectively represent the properties of nonlinear dynamic objects. The extrapolation properties of time-delayed neural networks deteriorate as the input signal amplitudes exceed the range covered by the used training set. The scientific novelty consists in determining a clear relationship between the types of input signals, their amplitudes, and the accuracy of the proposed models. The practical significance of investigation delineates the areas in which time-delay neural networks can be used to address the real-world challenges associated with significantly non-linear objects; demonstrates the increase in accuracy of identifying nonlinear objects compared to functional series models.

Keywords: identification, nonlinear objects, substantial nonlinearities, dynamic neural networks, simulation modeling.

Introduction

The development of technology and science makes it possible to ensure a qualitative increase in the characteristics of modern objects and processes in various fields of activity. On the other hand, this continuous process leads to the complexity of control objects, and tightening of requirements for their functioning (Agresti, 2017; Schoukens, & Ljung, 2019).

Today, in practical applications, dynamic control objects, which are characterized by significantly nonlinear properties are increasingly considered. Due to these characteristics, objects can function in more complex modes that cannot be realized using linear or weak nonlinear characteristics (Schoukens, & Ljung, 2019).

The most important from the point of view of practical use is the class of nonlinear dynamic objects with unknown operational laws and an unknown structure of the "black box" type (Rudin, & Radin, 2019). Such objects, as a rule, are characterized by a complete lack of a priori information about both the operational laws of the object, which can be estimated by methods of parametric identification, and its structure. For an external observer to study the object, only the reaction of the object $y(t)$ (output signal) to the test impact $x(t)$ (input signal) obtained as a result of the so-called "input-output" experiment is available (Rudin, & Radin, 2019; Todorovic, & Klan, 2006).

For successful interaction with such objects (solving control, management, and diagnostic tasks), it is first of all necessary to provide their adequate mathematical support and effective modeling tools. This leads to the use of nonparametric identification methods to build integral nonparametric dynamic models of the control objects. Such models are able to simultaneously take into account the nonlinear and dynamic properties of the object, which ensures high accuracy of model identification (Pavlenko, & Pavlenko, 2023).

It is also known that for a wide class of nonlinear dynamic objects, the relationship between the input $x(t)$ and the output signal $y(t)$ can be explicitly represented by the integral-power series (Volterra series) (Pavlenko, & Pavlenko, 2023). However, Volterra series also have a number of disadvantages that are significant for the above requirements for modern mathematical software and modeling tools. Thus, Volterra series are more suitable for modeling objects with weak smooth unambiguous nonlinearity. The presence of ambiguous or piecewise continuous nonlinear characteristics makes the use of integral-power series ineffective. In addition, the existing restrictions on the amplitudes of the input test signals also prevent the use of such models in the identification of control objects.

An alternative approach to modeling nonlinear dynamic objects is the use of artificial neural networks (NN) (Rao, & Reimherr, 2021; Liu *et al.*, A review, 2016). For a long time, their spread was significantly constrained by the lack of effective algorithmic and instrumental support, and the complexity of their interpretation (Fomin *et al.*, 2023). However, as a result of the progress of intelligent data processing technologies, today we can observe a noticeable increase in interest in NN in the direction of structural and functional identification of complex control object. At the same time, the models are mainly in the form of nonlinear static dependencies ((Rao, & Reimherr, 2021; Liu *et al.*, A review, 2016; Fomin *et al.*, 2023). Such models do not reflect all the properties of a real object, so they cannot provide high identification accuracy.

The literature also describes neural network structures capable of modeling a class of dynamic objects with nonlinear characteristics. However, their capabilities and areas of effective application are not sufficiently studied. This class of objects is inherent in many fields of activity (medicine, engineering, transportation, etc.) that is considered in this paper.

The aim of this paper is to improve the accuracy of modeling dynamic objects with significant nonlinearities using neural network models and to determine the scope of effective application of these models.

This goal is achieved by using dynamic nonlinear models in the form of time delay neural networks.

To study the area of effective application of the proposed models, the following tasks have been set.

1. Investigation of the accuracy of modeling nonlinear objects with smooth nonlinearity.
2. Investigation of the accuracy of modeling nonlinear objects with piecewise linear nonlinearity (saturation).

Literature review

Today, several methods are known for modeling nonlinear dynamic objects using NN (Wang *et al.*, 2009; Liu *et al.*, 2021): Dynamic Neuro-SM (Zhu *et al.*, 2016; Liu *et al.*, An overview, 2016), Wiener-type DNN (Liu *et al.*, 2022; Liu *et al.*, A Wiener-type, 2020; Liu *et al.*, 2017), and time-delay neural networks (TDNN) (Sugiyama *et al.*, 1991; Liu *et al.*, A time delay, 2020; Stegmayer *et al.*, 2004).

Dynamic Neuro-SM models are improved compared to the well-known static neuro-spatial mapping models (Zhu *et al.*, 2016; Liu *et al.*, An overview, 2016), which aim to map a given coarse model of an object to an accurate model. In Dynamic Neuro-SM models, neural networks are used to automatically match and change an existing coarse model into an accurate model through a learning process (Liu *et al.*, An overview, 2016; Sen, 2021). Such models provide an increase in accuracy compared to static neurospatial mapping models, but require some a priori information about the laws of functioning of the object of study (Liu *et al.*, An overview, 2016).

Wiener-type DNNs are based on the principle of constructing a nonlinear Wiener dynamic system, which consists of a simplified linear dynamic model followed by a nonlinear static model (Liu *et al.*, A Wiener-type, 2020; Liu *et al.*, A Wiener-type, 2017). The dynamic characteristics are mostly associated with the linear subsystem, while the nonlinear properties are contained only in the static nonlinear subsystem, which is implemented as an NN (Liu *et al.*, 2022; Liu *et al.*, A Wiener-type, 2020; Liu *et al.*, A Wiener-type, 2017). Such a structure can significantly increase the reliability of a dynamic neural model, but it has a complex (hybrid) structure, which imposes additional requirements on network training algorithms and narrows the scope of the model.

Among these model variants, TDNN are the most common structure consisting of several layers with direct signal propagation (Liu *et al.*, A time delay, 2020; Stegmayer *et al.*, 2004). Such models are capable of learning from the input-output data of nonlinear dynamic objects and have excellent convergence properties (Sen, 2021; Khandani and Mikhael, 2020), which are advantages over the aforementioned Dynamic Neuro-SM and Wiener-type DNN methods.

Due to their simplicity and versatility in modeling nonlinear dynamic objects, TDNN models are the most widely used.

The literature is quite full of studies of TDNN models for dynamic objects with weak nonlinearity ((Liu *et al.*, A time delay, 2020; Stegmayer *et al.*, 2004; Khandani and Mikhael, 2020). It is worth noting a number of publications devoted to the interpretation of TDNN, in particular, the establishment of an information link between these models and Volterra series (Fomin *et al.*, 2023; Stegmayer *et al.*, 2004). However, studies of models of dynamic objects with significant nonlinearity based on TDNN are not sufficiently reflected, which is a problem for the use of such models in applied problems.

This work is aimed at filling the identified gap and focuses on the study of NN for modeling dynamic objects with nonlinear characteristics, identifying the scope of their effective application in solving applied problems of identifying objects with significantly nonlinear characteristics.

Dynamic models based on neural networks with time delay

TDNN models are an effective tool for modeling nonlinear dynamic objects with continuous characteristics. The most commonly used TDNN structure consists of three layers: input, hidden, and output.

There are many structures of neural networks: with several hidden layers, different activation functions, and topologies. However, the use of these structures results in a more complex expression for the output of the NN. This is a significant disadvantage compared to three-layer TDNN for modeling nonlinear dynamic objects.

In the above structure, the input layer of a TDNN includes M neurons, where M is the memory length of the object model. The number of neurons M is chosen to best reflect the dynamic properties of the object. The input layer receives the data $\mathbf{x}(t_n)=[x(t_n), x(t_{n-1}), \dots, x(t_{n-M+1})]$, $t_n=n\Delta t$, $n=1, 2, \dots$

The hidden layer includes K neurons with a nonlinear activation function. The number of neurons K is chosen to best reflect the nonlinear properties of the object.

The TDNN output layer includes 1 neuron with a linear activation function. The signal $y(t_n)$ at the output layer at time t_n depends on the values of the input signal $\mathbf{x}(t_n)$ and is determined by the expression (Giannini *et al.*, 2007):

$$y(t_n) = b_0 + S_0 \sum_{i=1}^K w_i S_i \left(b_i + \sum_{j=1}^M w_{i,j} x(t_{n-j}) \right) \quad (1)$$

where b_0 , b_i are the biases of the neurons of the output and hidden layers, respectively; S_0 , S_i are the activation functions of the neurons of the output and hidden layers, respectively; w_i , $w_{i,j}$ are the weighting coefficients of the neurons of the output and hidden layers, respectively.

Fig. 1 shows the TDNN architecture as a three-layer feedforward network with M inputs, a hidden layer with K neurons, and one output neuron. Such a model can be trained to dynamic behavior with nonlinear characteristics based on input-output data.

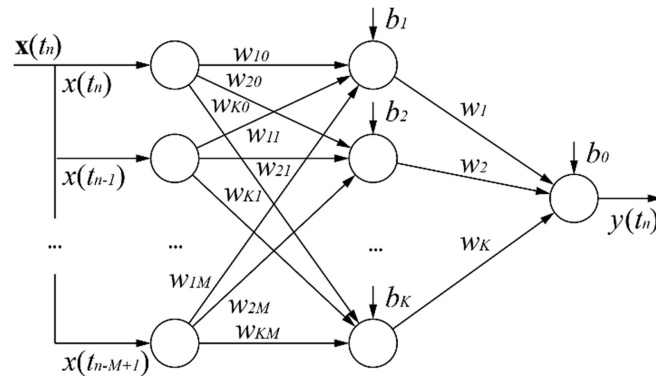


Figure 1. TDNN architecture as a three-layer network with direct signal propagation

Source: developed by the author on the basis of researches (Stegmayer *et al.*, 2004; Giannini *et al.*, 2007)

Setting up the experiment

A simulation model of a test object

The accuracy of TDNN models is studied using the example of a test object. The simulation model of the test object with a first-order dynamic block and a nonlinear feedback block (Fomin *et al.*, 2023) is shown in Fig. 2.

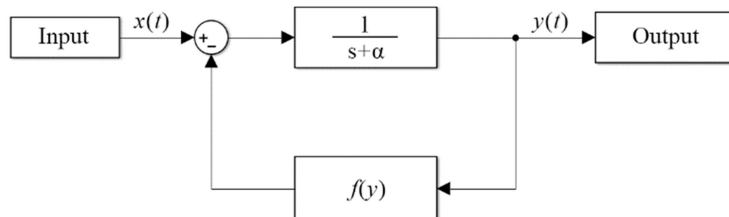


Figure 2. Simulation model of the test nonlinear dynamic object

Source: developed by the author on the basis of research (Fomin *et al.*, 2023)

The feedback block uses a nonlinear function as $f(y)$. In this case, 2 types of nonlinearity are considered.

1. For the case of smooth nonlinearity, it is polynomial: $f_1(y) = \beta y^2$, where β is a parameter (constant).

2. For the case of significant nonlinearity, a function with saturation:

$$f_2(y) = \begin{cases} s, & y > p \\ k \cdot y, & |y| \leq p \\ -s, & y < -p \end{cases} \quad (2)$$

where s is the saturation level, p is the saturation start point, and $k=s/p$ is the gain.

In order to study the accuracy of TDNN models of a test object with different types of nonlinearity $f_i(y)$ ($i=1,2$), training $\mathbf{x}_i^{train}(t_n)$ and test $\mathbf{x}_i^{test}(t_n)$ data sets are generated based on an input/output experiment. To generate these data, test signals $x(t)$ in the form of pulse, step, linear, and harmonic functions with different amplitudes a are fed to the input of the simulation model.

When performing the input/output experiment with different types of nonlinearity, the following parameters of the simulation model were adopted: $\alpha=2.64$; for the case of smooth nonlinearity, $\beta=1.45$; for the case of significant nonlinearity, $s=0.7$, $p=0.7$, $k=1$.

Modeling tools

The Keras library (keras.io) is used for the programmatic implementation of the neural network. This is one of the most popular Python libraries for building small networks with a sequential structure, where layers follow each other, with one input and one output layer.

To build a direct propagation network with Keras, you can use any number of consecutive layers of predefined types: Input, Dense, and Activation. The library has a ready-made set of loss functions and optimization algorithms that allow you to quickly train models and avoid local minima.

A three-layer neural network was created using the Keras library. The input layer consists of M neurons that are fed with the data $\mathbf{x}(t_n)$. The hidden layer consists of K neurons. The output layer consists of one neuron. The block diagram of the TDNN model created using the Keras library is shown in Fig. 3.

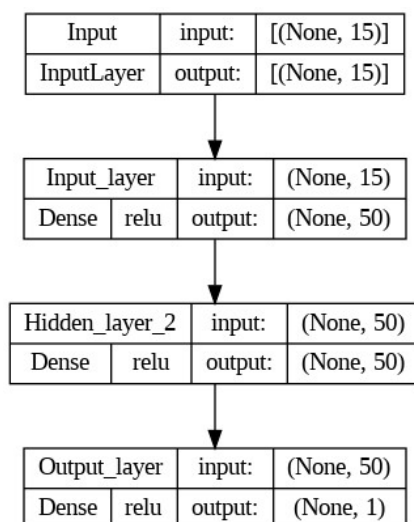


Figure 3. Block diagram of TDNN as a three-layer network with direct signal propagation

Notes: *None* in the dimension vectors of the input and output data of each layer means a variable number of rows in the data.

Source: developed by the author on the basis of Keras library

Defining TDNN parameters

To determine the best values of M and K in a given TDNN structure, a number of neural networks with different numbers of neurons in the input and hidden layers are built.

The result of the experiment as a function of the number of neurons in the input and hidden layers is shown in Fig. 4. The root mean square error function mse from the Keras library is used as a loss.

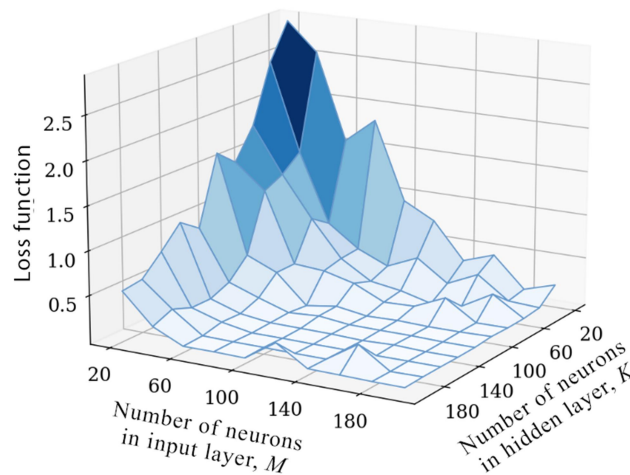


Figure 4. Dependence of losses on the number of neurons in the input and hidden layers

Source: developed by the author

The result of the experiment as a function of the learning time (epoch) and the number of neurons in the input and hidden layers is shown in Fig. 5.

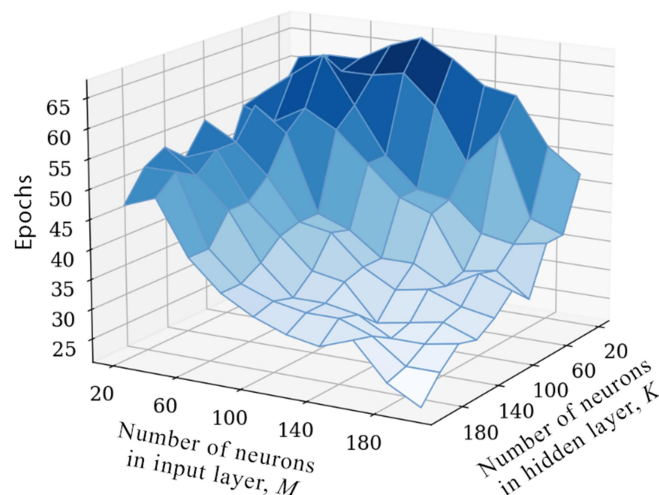


Figure 5. Dependence of training time on the number of neurons in the input and hidden layers

Source: developed by the author

As a result of comparing Figures 4 and 5, the values of the number of neurons in the input and hidden layers of the TDNN $M=15$ and $K=50$, respectively, were chosen to ensure the level of losses ($loss < 1.0$) set by the experimental conditions with an acceptable training time ($epochs < 40$). The obtained TDNN was used to study the accuracy of models of dynamic objects with smooth and significant nonlinearities.

Study of models of dynamic objects with smooth and significant nonlinearities

To study the accuracy of modeling dynamic objects with smooth and significant nonlinearities using TDNN models and determine the scope of their effective application, two experiments were organized and conducted:

1. Study of the scalability of TDNN models to different input signals $x(t)$.
2. Study of extrapolation properties of TDNN models.

The results of both experiments are compared with the results of modeling using deterministic identification methods, namely, integral-power series based on multivariate weight functions.

The models $y_v(t)$ in the form of integral-power series based on multidimensional weight functions are constructed for the test objects by the compensation method (Pavlenko, & Pavlenko, 2023). Fig. 6 shows the $y_v(t)$ models obtained for test dynamic objects with smooth $f_1(y)$ (Fig. 6a) and significant $f_2(y)$ (Fig. 6b) nonlinearities. The 1st-order multivariate weight function $w_1(t)$ and the diagonal cross section of the 2nd-order multivariate weight function $w_2(t, t)$ are also shown here, as well as the object responses $y_1(t)$ and $y_2(t)$ to the input signals $x_1(t) = a\delta(t)$ and $x_2(t) = 2a\delta(t)$ ($a = 0.5$), which are used to identify the multivariate weight functions.

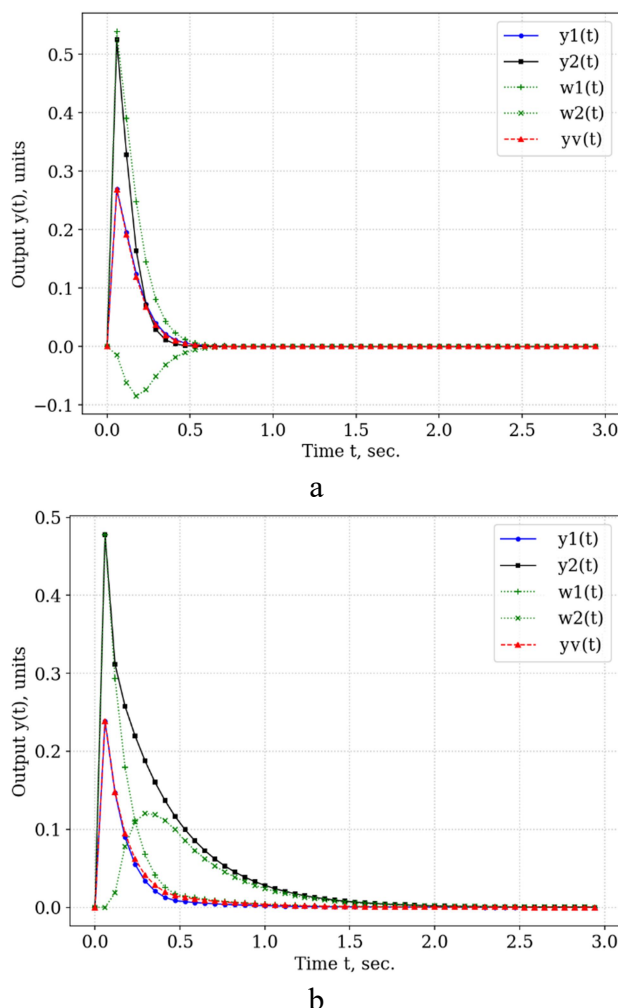


Figure 6. Models of $y_v(t)$ in the form of integral-power series based on multidimensional weight functions obtained for test dynamic objects

Notes: a – smooth $f_1(y)$ nonlinearity, b – significant $f_2(y)$ nonlinearity.

Source: developed by the author

Study of the scalability of TDNN models to different input signals.

The training dataset $\mathbf{x}_i^{train}(t_n)$ is formed on the basis of pulse signals $x(t) = a\delta(t)$ of different amplitudes ($a \in (0, 1]$) at the input of the object and responses $y(t)$ at its output.

The test dataset $\mathbf{x}_i^{test}(t_n)$ includes step $x(t)=a\Theta(t)$, linear $x(t)=at$, and harmonic $x(t)=asin(t)$ signals of different amplitudes ($a \in (0, 1]$) at the input of the object and responses $y(t)$ at its output.

The experiment is performed for objects with nonlinearities in the feedback block in the form of a polynomial function $f_1(y)$, as well as a saturated function $f_2(y)$.

The TDNN model is built on the data of the training set $\mathbf{x}_i^{train}(t_n)$. The scalability of the obtained TDNN model to different input signals is studied on the data of the test set $\mathbf{x}_i^{test}(t_n)$.

The output of the TDNN model $y_{nn}(t)$ is compared with the output of the simulation model $y(t)$ and the result of identifying $y_v(t)$ as an integral-power series based on multidimensional weight functions (Pavlenko, & Pavlenko, 2023; Antipina *et al.*, 2023).

Experiment 1. We investigated the accuracy of modeling using TDNN and integral-power models under the influence of input signals $x(t) = a\delta(t)$ of different amplitudes ($a \in (0, 1]$). Fig. 7 shows a comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of the signal $x(t)=a\delta(t)$ ($a=0.65$) at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively.

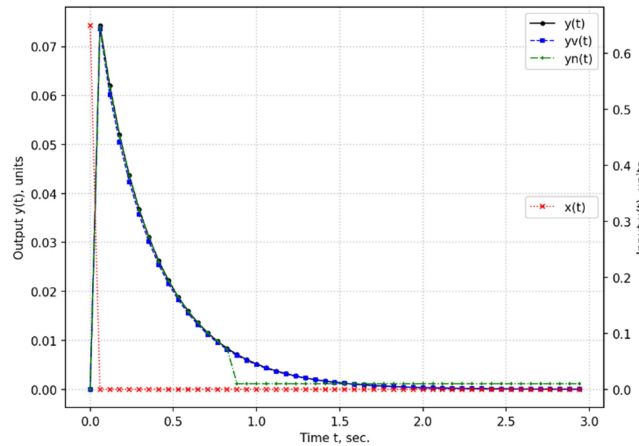


Figure 7. Comparison of the output signals $y_{nn}(t)$, $y_v(t)$ and $y(t)$ obtained as a result of the action of the signal $x(t)=a\delta(t)$ ($a=0.65$) at the inputs of the TDNN model, integral power series and simulation model of a nonlinear dynamic object, respectively
Source: developed by the author

Experiment 1 demonstrates the comparable accuracy of modeling using TDNN and integral-power models under the input signals $x(t)=a\delta(t)$.

Experiment 2. We investigated the accuracy of modeling using TDNN and integral-power models under the influence of input signals $x(t)=a\Theta(t)$, $x(t)=at$ and $x(t)=asin(t)$ of different amplitudes ($a \in (0, 1]$). In Fig. 8 shows a comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of the signals $x(t)=a\Theta(t)$ (Fig. 8a), $x(t)=at$ (Fig. 8b), and $x(t)=asin(t)$ (Fig. 8c), $a=0.65$, at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively.

The experiment demonstrates that the TDNN model is significantly inferior in accuracy to the integral-power model under the influence of the input signals $x(t)=a\Theta(t)$, $x(t)=at$, and $x(t)=asin(t)$, which were not included in the training dataset $\mathbf{x}_i^{train}(t_n)$.

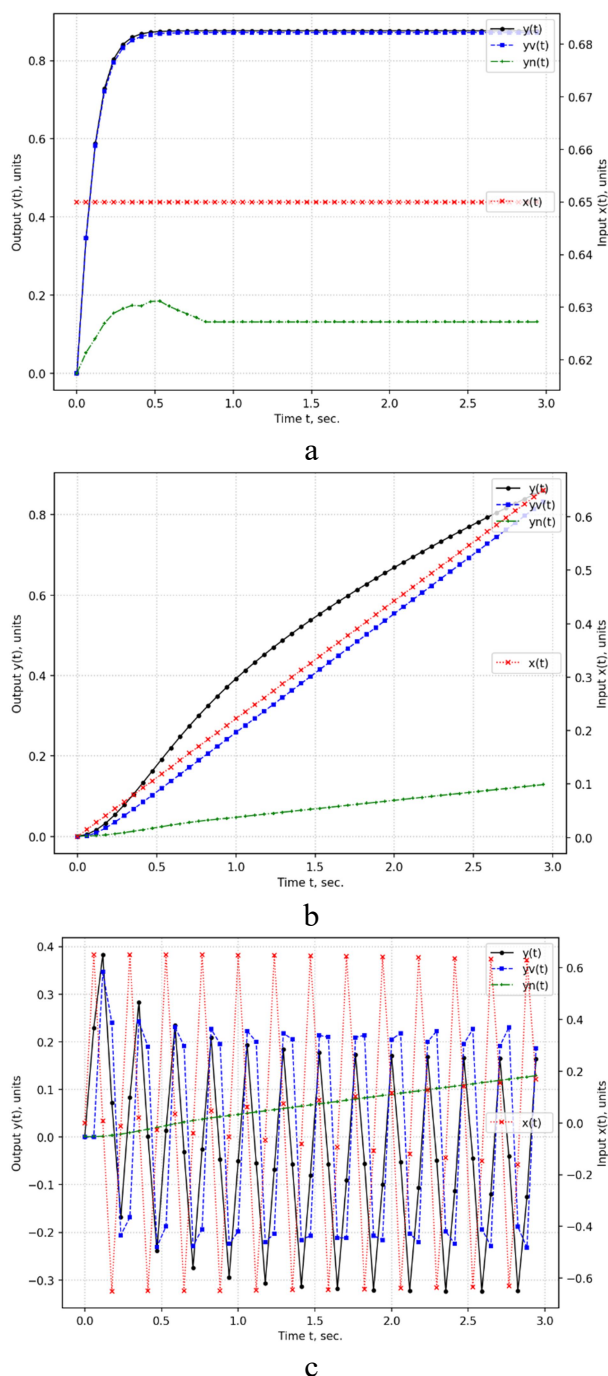


Figure 8. Comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of signals at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively

Notes: a – $x(t) = a\Theta(t)$; b – $x(t) = at$, c – $x(t) = a\sin(t)$, $a = 0.65$

Source: developed by the author

The experiment shows that TDNN models are not invariant to the shape of the input signal. A TDNN model can adequately reflect the properties of a dynamic object when trained on a sufficient amount of data. The training dataset $\mathbf{x}_i^{train}(t_n)$ should be formed on the basis of input signals of different amplitudes of the same type as in the test dataset $\mathbf{x}_i^{test}(t_n)$. This is a disadvantage of neural network models in comparison with models based on integral-power

series based on multidimensional weight functions (Pavlenko, & Pavlenko, 2023; Antipina *et al.*, 2023; Mitrea *et al.*, 2009; Meruelo *et al.*, 2016).

Study of interpolation and extrapolation properties of TDNN models.

The training dataset $\mathbf{x}_i^{train}(t_n)$ is formed on the basis of pulse $x(t) = a\delta(t)$, step $x(t) = a\Theta(t)$, linear $x(t) = at$, and harmonic $x(t) = asin(t)$ signals of different amplitudes ($a \in (0, 1]$) at the input of the object and responses $y(t)$ at its output.

The test dataset $\mathbf{x}_i^{test}(t_n)$ includes the same signals $x(t)$ with different amplitudes ($a \in (1, 2]$) at the input of the object and responses $y(t)$ at its output.

The experiment is performed for objects with nonlinearities in the feedback block in the form of a polynomial function $f_1(y)$, as well as a saturated function $f_2(y)$.

The TDNN model is built on the data of the training set $\mathbf{x}_i^{train}(t_n)$. The study of the extrapolation properties of the obtained TDNN model is performed on the data of the test set $\mathbf{x}_i^{test}(t_n)$.

The output of the TDNN model $y_{nn}(t)$ is compared with the output of the simulation model $y(t)$ and the result of identifying $y_v(t)$ as an integral-power series based on multidimensional weight functions (Pavlenko, & Pavlenko, 2023; Antipina *et al.*, 2023).

Experiment 3. We investigated the accuracy of modeling using TDNN and integral-power models under the influence of pulsed, stepped, linear, and harmonic input signals of different amplitudes ($a \in [0.1, 0.2, \dots, 1.0]$). Fig. 9 shows a comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of the signal $x(t)=a\Theta(t)$ ($a=0.7$) at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively.

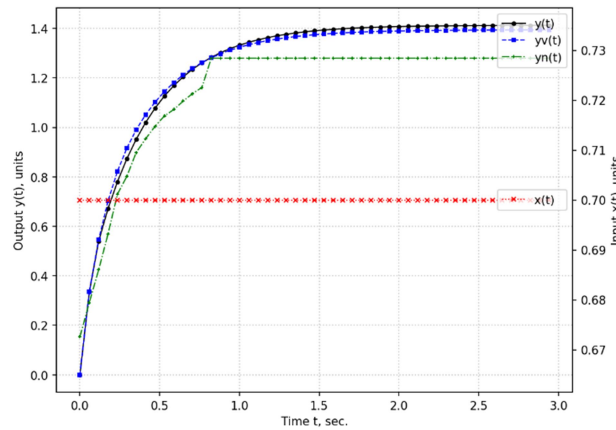


Figure 9. Comparison of the output signals $y_{nn}(t)$, $y_v(t)$ and $y(t)$ obtained as a result of the action of the signal $x(t)=a\Theta(t)$ ($a=0.7$) at the inputs of the TDNN model, integral power series and simulation model of a nonlinear dynamic object, respectively

Source: developed by the author

As a result of Experiment 3, we obtained the mean square error values for TDNN models compared to integral-power series based on multivariate weighting functions for the nonlinear functions $f_1(y)$ and $f_2(y)$, which are shown in Table 1.

Experiment 3 demonstrates the comparable accuracy of modeling using TDNN and integral-power models of the test object with smooth nonlinearity $f_1(y)$. However, when identifying a test object with a significant nonlinearity $f_2(y)$, TDNN models significantly outperform the integral-power models in terms of accuracy.

Table 1. Mean square error of the test object modeling ($a=0.7$)

Type of nonlinearity	Input signal $x(t)$							
	impulse $a\delta(t)$		step $a\Theta(t)$		linear at		harmonic $asin(t)$	
	$y_v(t)$	$y_{nn}(t)$	$y_v(t)$	$y_{nn}(t)$	$y_v(t)$	$y_{nn}(t)$	$y_v(t)$	$y_{nn}(t)$
Polynomial	0,074	0,080	0,082	0,087	0,088	0,096	0,103	0,127
With saturation	0,152	0,121	0,167	0,133	0,209	0,139	0,264	0,184

Source: developed by the author

Experiment 4. We investigated the interpolation and extrapolation properties of models using TDNN and integral-power models under the influence of pulsed, stepped, linear, and harmonic input signals of different amplitudes:

- for the case of studying the interpolation properties of models $a \in [0.05, 0.15, \dots, 0.95]$;
- for the case of studying the extrapolation properties of models $a \in [1.05, 1.15, \dots, 1.95]$.

In Fig. 10 shows a comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of the signal $x(t)=a\Theta(t)$ at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively, for the interpolation task at $a = 0.65$ (Fig. 10a) and the extrapolation task at $a = 1.65$ (Fig. 10b).

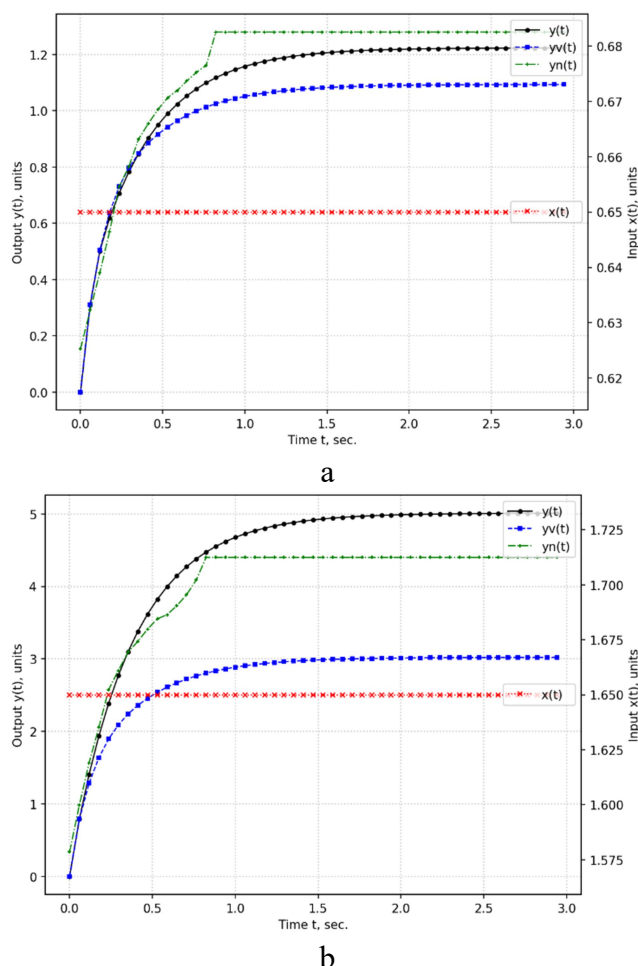


Figure 10. Comparison of the output signals $y_{nn}(t)$, $y_v(t)$, and $y(t)$ obtained as a result of the action of the signal $x(t)=a\Theta(t)$ at the inputs of the TDNN model, the integral-power series, and the simulation model of a nonlinear dynamic object, respectively

Notes: a – interpolation problem ($a=0.65$); b – extrapolation problem ($a=1.65$)

Source: developed by the author

Experiment 4 demonstrates comparable modeling accuracy in the case of studying the interpolation properties of TDNN and integral-power models.

The extrapolation properties of TDNN models deteriorated with increasing input signal amplitudes beyond the range of input signal amplitudes of the training set $\mathbf{x}_i^{train}(t_n)$. Thus, using the input signal $x(t)=1.65\Theta(t)$, the accuracy of the TDNN model decreases by 25%.

At the same time, the accuracy of the model in the form of an integral-power series based on multidimensional weight functions in the case of using a significantly nonlinear function $f_2(y)$ is 30% lower than the TDNN model.

Thus, the area of effective application of TDNN models is the identification of objects with significantly nonlinear characteristics.

Results and discussion

The obtained modeling results show that TDNN models are not invariant to the signal received at the input of the object, which is a disadvantage of these models. However, TDNN models can adequately reflect the properties of nonlinear dynamic objects when trained on a sufficient amount of data generated from input signals of the same type as in the test data set.

The extrapolation properties of TDNN models will deteriorate with increasing input signal amplitudes that go beyond the range of input signal amplitudes of the training set.

When identifying objects with significantly nonlinear properties, such as a saturation function, TDNN models are 10-25% more accurate than integral-power series based on multivariate weight functions.

Thus, the area of effective application of TDNN is the identification of objects with significantly nonlinear properties.

Conclusions

The paper considers the approach to modeling of dynamic objects with essential nonlinearities on the basis of time-delay neural networks. Experiments are organized and researches are carried out to determine the accuracy of the proposed models and the area of effective application of these models.

It is experimentally confirmed that the use of the proposed models for identification of dynamic objects with essentially nonlinear characteristics allows to increase the accuracy of the modeling process by 10-25% in comparison with the models based on deterministic methods of identification, such as integral-power series. based on multidimensional weight functions.

Thus, the area of effective application of models in the form of time-delay neural networks has been established. It is modeling of dynamic objects with significant nonlinearities, where traditional identification methods don't provide sufficient modeling accuracy.

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Conflict of Interest

The authors have no conflict of interest.

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МОДЕЛІ ДИНАМІЧНИХ ОБ'ЄКТІВ ЗІ ЗНАЧНОЮ НЕЛІНІЙНІСТЮ НА ОСНОВІ НЕЙРОННИХ МЕРЕЖ ІЗ ЧАСОВИМИ ЗАТРИМКАМИ

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Анотація. Робота присвячена проблемі нелінійного моделювання об'єктів на основі динамічних нейронних мереж. Метою роботи є підвищення точності моделювання динамічних об'єктів зі значними нелінійностями за допомогою нейромережових моделей та визначення області ефективного застосування цих моделей. Ця мета досягається шляхом застосування нелінійних динамічних моделей у вигляді нейронних мереж із часовою затримкою. Для дослідження області ефективного застосування запропонованих нейромережових моделей розглядаються тестові об'єкти з нелінійностями двох типів: гладкою та кусково-лінійною (насиченням). Для дослідження точності нейронних мереж із часовою затримкою при моделюванні нелінійних динамічних об'єктів проведено два експерименти: дослідження масштабованості моделей до різних вхідних сигналів; дослідження екстраполяційних властивостей моделей. Результати обох експериментів порівнюються з результатами моделювання за допомогою компенсаційного методу детермінованої ідентифікації у вигляді функціональних рядів на основі багатовимірних вагових функцій. Отримані результати моделювання свідчать, що запропоновані нейромережові моделі не є інваріантними щодо вхідного сигналу. Однак ці моделі можуть адекватно відображати властивості нелінійних динамічних об'єктів в разі навчання на достатньому обсязі даних, що формується на основі вхідних сигналів того ж типу, що й у тестовому наборі даних. Екстраполяційні властивості

нейронних мереж із часовою затримкою погіршуються зі збільшенням амплітуд вхідних сигналів, що виходять за межі діапазону амплітуд вхідних сигналів навчальної вибірки. Наукова новизна роботи полягає у визначенні залежності між типами сигналів та їх амплітудами, що діють на вході моделі, і точністю запропонованих моделей. Практична користь роботи полягає у визначенні області ефективного застосування нейронних мереж із часовою затримкою під час розв'язування прикладних задач ідентифікації об'єктів зі значно нелінійними характеристиками; підвищенні точності ідентифікації нелінійних об'єктів порівняно з моделями у вигляді функціональних рядів на основі багатовимірних вагових функцій.

Ключові слова: ідентифікація, нелінійні об'єкти, суттєві нелінійності, динамічні нейронні мережі, імітаційне моделювання.